Estimates and orders of magnitude

1. The density of lead is $11.3 g/cm^3$. What is this value in kilograms per cubic meter?
2. How many kernels of corn does it take to fill a 2 litre soft drink bottle?
3. What total volume of air does a person breathe in a lifetime? Estimate that a person breathes about $500 cm^3$ of air with each breath.
4. How many times does a human heart beat during a lifetime? How many litres of blood does it pump? Estimate that the heart pumps $50 cm^3$ of blood with each beat.
5. How many drops of water are in all the oceans on earth?
6. How many dollar bills would have to be stacked to reach the moon? Would that be cheaper than building and launching a spacecraft?

Vectors

7. Identify vectors and scalars among the following physical quantities:
   time, density, mass, surface, displacement, length (path travelled), velocity, speed, acceleration, force, linear momentum, work, temperature.
8. A person walks along a semicircle of radius $R = 20m$**.** Find the displacement vector and its length. Consider the case of the full circle.
9. Given two vectors **A** i **B** such that: **A**+**B**=11**i**–**j**+5**k** and **A**–5**B**= –5**i**+10**j**+15**k**. Find **A** and **B**.
10. Find the angle between the two vectors $\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{B} = -4\hat{i} + 2\hat{j} - \hat{k}$.
11. Given vector **a**=7**i** +11**j.** Find a unit vector perpendicular to the vector **a.**
12. Given vectors: $\vec{A} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ oraz $\vec{B} = \hat{i} - 2\hat{j} + 7\hat{k}$. Find: a) the length of each vector, b) the dot product (scalar product) $\vec{A} \cdot \vec{B}$**,** c) the vector product $\vec{A} \times \vec{B}$**,** d) the angle between the vectors $(\vec{A} - \vec{B})$ and $(\vec{A} + \vec{B})$.
13. Providing that $|\vec{A} \times \vec{B}| = \vec{A} \cdot \vec{B}$ what is the angle between the vectors $\vec{A}$ i $\vec{B}$?
14. Does exist a vector $\vec{A}$ such that $(2\hat{i} - 3\hat{j} + 4\hat{k}) \times \vec{A} = (4\hat{i} + 3\hat{j} - \hat{k})$? How reasonably quickly check it out?
15. Given two vectors **a**=3**i**+4**j** and **b**=6**i**+16**j**. Find components of the vector **b** parallel and perpendicular to the vector **a.**
16. When two vectors $\vec{A}$ and $\vec{B}$ are drawn from a common point, the angle between them is α. Using vector techniques, show that the magnitude of their vector sum is given by $\sqrt{A^2 + B^2 + 2AB\cos\alpha}$ (*the law of cosines*).
17. The vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, called the position vector, points from the origin (0,0,0) to an arbitrary point in space with coordinates $(x, y, z,)$. Show, that all the points $(x, y, z,)$ that satisfy the equation $Ax + By + Cz = 0$, where A, B, C are constants, lie in a plane that passes through the origin and that is perpendicular to the vector $A\hat{i} + B\hat{j} + C\hat{k}$. Sketch this vector and the plane.

18. Find the derivatives (with respect to *x*) of the following functions: $x^3 \sin(x)$, $\sqrt{3x}$, e^{x^2+3}, $\sin(y^2)$, $\cos^4(x)$, $tg(x) \equiv \dfrac{\sin(x)}{\cos(x)}$, $\ln(x^2)$, $xy\sin(z)$, $f^2(x)$, $\sin(\omega x)$.